"DIVIDE THE LAND EQUALLY" (Ezekiel 47:14)

Fair Division of Electricity

Research program

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Image credit: https://www.weforum.org/projects/systems-of-cyber-resilience-electricity

Motivation

- In developing countries, total demand for electricity is larger than the supply.
- Solution: **load shedding**.

- Common practice: disconnecting entire regions.

Motivation

- Current technology allows to disconnect *individual houses*.
- Challenge: take *individual preferences* into account.
- Baseline work:
  Oluwasuji, Malik, Zhang, Ramchurn:
  "*Solving the fair electric load shedding problem in developing countries*".
  IJCAI ’18, JAAMAS ’20.

Model

INPUT:

- Total supply in kilowatts = S.
- For each agent i in $1,\ldots,n$:
  - size z_i in kilowatts.
  - utility $u_i(t)$ of being connected at t.

OUTPUT:

- For each time t, subset $A(t)$ of agents.
- $A(t)$ is *feasible*: **Sum** $_{i \text{ in } A(t)}$ $\boldsymbol{z_i} \leq \boldsymbol{S}$.

Cases

1. Identical sizes, uniform* utilities.
   (agents only care how much time they are connected).*

2. Identical sizes, additive** utilities.
   *(** utility of agent i = integral of $u_i(t)$ over connected time).*

3. Different sizes, uniform utilities.

4. Different sizes, additive utilities.

1. Identical sizes, uniform utilities

- q := floor(S/z) = #agents in every maximal feasible set.
- Connect q agents each time.
- Connect each agent q/n of the time.
- Utility per agent: q/n of total utility.

2. Identical sizes, additive utilities

- q := floor(S/z).

- Construct a “cake” from q adjacent copies of the timeline of a week:

- Divide the “cake” using any algorithm for *proportional cake-cutting*.
- Utility per agent:
  - $\geq 1/n$ of total cake utility.
  - $\geq q/n$ of total week utility.

3. Different sizes, uniform utilities

Example: n=3 agents; S=30 [kW];
sizes 10, 20, 30 [kW].

- Maximal feasible sets: {10,20} ; {30}.
- What is a *fair* allocation of time?

3. Different sizes, uniform utilities: *Fractional approval voting*

Fractional approval voting:

- INPUT: each of *n* agents approves one or more candidates.
- OUTPUT: each candidate is elected for a fraction of the time.

Electricity division:

- Candidates = maximal feasible sets.
- Agent *i* approves all sets containing *i*.

3. Different sizes, uniform utilities: *Fractional approval voting*

Equivalent terms:

- Bogomolnaia, Moulin, Stong (2005):
  Collective choice under dichotomous preferences.

- Duddy (2015):
  Fair sharing under dichotomous preferences.

- Aziz, Bogomolnaia, Moulin (2019):
  Fair mixing: the case of dichotomous preferences.

- Brandl, Brandt, Peters, Stricker (2021):
  Distribution rules under dichotomous preferences.

3. Different sizes, uniform utilities: *Fractional approval voting*

Main fairness notions – Fair Share (FS):

Individual FS: each agent gets utility ≥ $1/n$.

Egalitarian: each agent gets ≥ r; maximize r (given the sizes)

1/2{10,20} + 1/2*{30}*

Group FS: For each group of q agents: total allotment to candidates approved by at least 1 member ≥ q/n.

2/3{e,30-e} + 1/3*{30}*

3. Different sizes, uniform utilities: *Egalitarian allocation*

Goal: Maximize r such that each agent can be connected at least r of the time.

- NP-hard (reduction from PARTITION).
  - # candidates is exponential in n.
- **Bin-packing lower bound**:
  - Pack agents into k bins of size S.
  - Connect each bin $1/k$ of the time.
- Not optimal. Example: S=2, sizes=1,1,1.
  - $k = 2$; lower bound = 1/2;
  - But $r = q/n = 2/3$ is possible.

3. Different sizes, uniform utilities: *Egalitarian allocation*

***q*-times bin-packing lower bound**:

- Pack agents into *k bins* of size *S*; *put each agent in q different bins*.
- Connect each bin 1/k of the time.
- Egalitarian value ≥ *q*/k.

Research questions:

a) Can efficient bin-packing algorithms be adapted to *q*-times bin-packing?
b) Is the *q*-times bin-packing lower bound always tight for some *q* ≥ 1?

3. Different sizes, uniform utilities

Research questions:

a) What bin-packing algorithms can be adapted to q-times bin-packing?

b) Is the q-times bin-packing lower bound always tight for some $q \geq 1$?

c) What efficient algorithms can be used to find a group-FS allocation?

4. Different sizes, additive utilities

Question (a): Let *r* be the egalitarian value of the sizes with *uniform* utilities.

- Can we guarantee each agent utility ≥ *r* with *additive* utilities?

Answer: Yes: Assuming utilities in each minute are uniform - partition each minute by the egalitarian solution.

- Too many cuts.

4. Different sizes, additive utilities

Question (b): can we guarantee each agent utility ≥ r with *few cuts*?

Answer: If egalitarian value comes from q-times bin-packing with k *bins* ($r = q$/k):

- Find a ***k-consensus division*** - a partition into k pieces that each agent 1,..., n-1 values at exactly 1/k.
    - Requires ≤ (n-1)(k-1) cuts (Alon, 1987).
- Agent n pick q pieces for its bins (util≥q/k); every other bin gets arbitrary piece (util=q/k).

4. Different sizes, additive utilities: *Consensus division*

k-consensus division is hard even when *k*=2 and all valuations are piecewise-constant:

- Computing an approximate *k*-consensus division with *n*(*k*-1) cuts is PPA-hard (Filos-Ratsikas, Goldberg; 2018; Filos-Ratsikas, Hollender, Sotiraki, Zampetakis; 2020).
- Deciding whether there exists a *k*-consensus division with *n*(*k*-1)-1 cuts is NP-hard (Filos-Ratsikas, Frederiksen, Goldberg, Zhang; 2018).

Questions:

a) Can electricity division be done with *fewer* cuts?

b) Can electricity division be done *more efficiently*?

4. Different sizes, additive utilities: *Hardness*

Theorem. Egalitarian electricity division:

- May require n-1 cuts;
- May be PPA-hard to compute.

Proof. Given a *2*-consensus division problem with *n-1* agents with valuations $v_1,...,v_{n-1}$, construct an electricity division problem with n agents:

- n-1 agents: size $S/(n-1)$, valuations $v_1,...,v_{n-1}$.
- One agent: size S, valuation $(v_1+...+v_{n-1})/(n-1)$.

An egalitarian electricity division with value 1/2 === a consensus division among the n-1 agents. ***

4. Different sizes, additive utilities

Research questions:

a) Are *n*-1 cuts always sufficient for egalitarian electricity division?
b) How many cuts are needed for a group-FS division?
c) Can we heuristically find an egalitarian / group-FS division with few cuts on realistic instances?

Extensions of Basic Model

a) More complex supply constraints:
- S can change over time;
- Supply network with link capacities.

b) More complex demands:
- z_i can change over time;
- Prices can be used to incentivize agents to reduce their size.

We are recruiting post-doctoral students

Conclusion

Electricity division is related to several classic problems:

- **Proportional cake-cutting;**
- **Bin packing;**
- **Fractional approval voting;**
- **Consensus division.**

Ideas may be applicable to other settings of social choice or fair division in which agents may have *different sizes*.