A truthful Multi Item-Type Double-Auction Mechanism

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with
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Intro: one item-type, one unit

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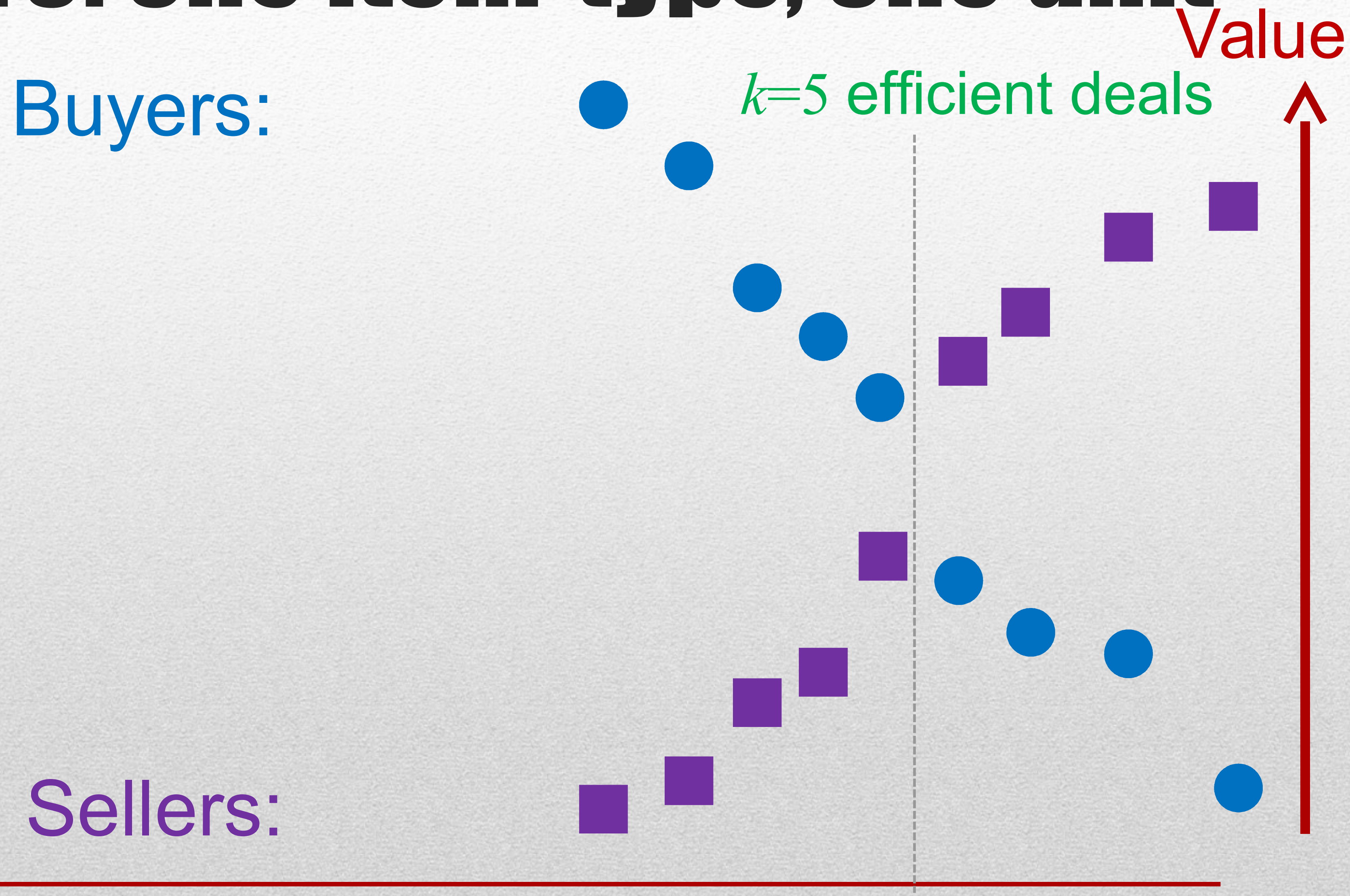
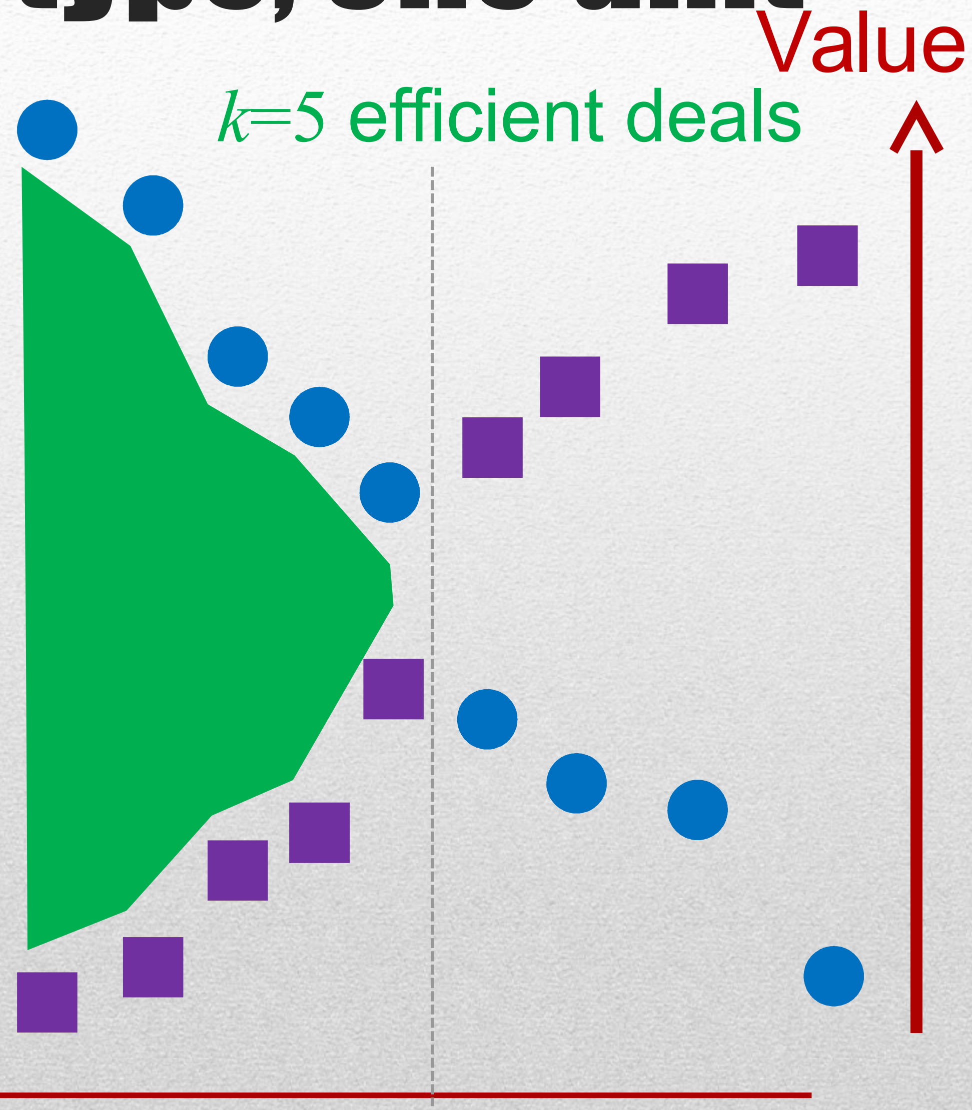


Intro: one item-type, one unit

Buyers:

Gain from trade:

Sellers:



Price Equilibrium

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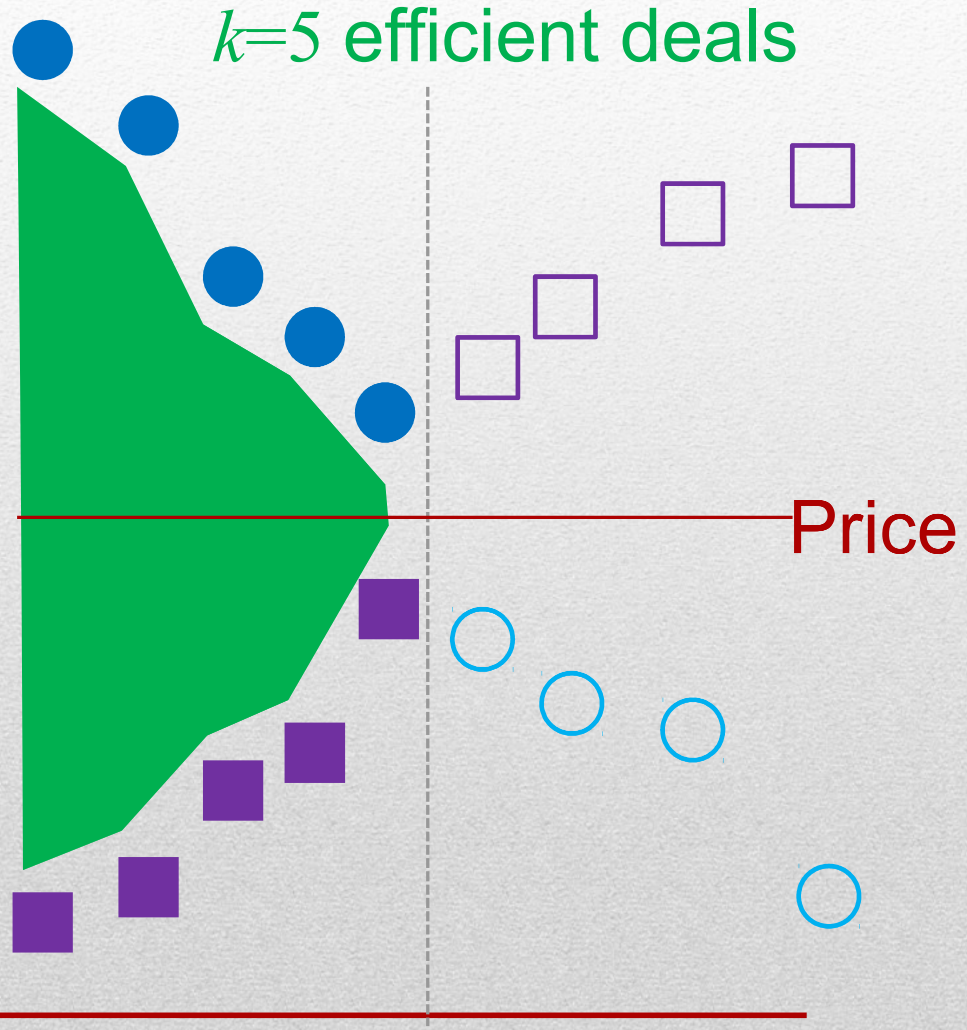
✓ Maximum gain



Price Equilibrium

- ✓ Maximum gain
- ✓ Handles traders with many item-types if they are *Gross-Substitutes*
  (= no complementarities)

Price Equilibrium

- ✓ Maximum gain
- ✓ Handles traders with many item-types if they are *Gross-Substitutes* *(= no complementarities)*
- ✗ Not truthful

Some related work

Bayesian prior:

- Single-sided auction: Myerson [1981], Blumrosen and Holenstein [2008], Segal [2003], Chawla et al. [2007-2010], Yan [2011].
- Double auction: Xu et al. [2010], Loertscher et al. [2014], Blumrosen and Dobzinski [2014], Colini-Baldeschi et al. [2016].

Prior-independent:

- Single-sided auction: Cole and Roughgarden [2014], Dhangwatnotai et al. [2015], Huang et al. [2015], Morgenstern and Roughgarden [2015], Devanur et al. 2011], Hsu et al. [2016].
- Double auction: Baliga and Vohra [2003] – single-parametric agents.

Prior-free:

- Single-sided auction: Goldberg et al. [2001-2006], Devanur et al. [2015], Balcan et al. [2007-2008]
- Double auction: **McAfee [1992]** →

McAfee (1992) (simplified)

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Extensions:
Babaioff et al. [2004-2006],
Gonen et al. [2007],
Duetting et al. [2014] –
Single-parametric agents.
Blumrosen & Dobzinsky [2014] -
Single item-type, Gain ~ 1/48.

Prior-Free Double-Auctions

	Tru	Gain	Agents
Equilibrium	No	1	Multi-parametric (Gross-substitute)
McAfee family	Yes	1-o(1)	Single-parametric / Single-item-type
Our goal	Yes	1-o(1)	Multi-parametric, multi-item-type

Prior-Free Double-Auctions

	Tru	Gain	Agents
Our goal	Yes	1-o(1)	Multi-item-type

Our current assumptions:

1. Buyers – at most g item-types, **gross-substitute**.
   Sellers – 1 item-type, **decreasing marginal gain**.
2. **Large market** – for each item-type x, $k_x \to \infty$;
   at most m units per seller;
3. **Bounded variability** – $k_{max} / k_{min} \le c$
4. **Generic valuations** – no ties.

MIDA: Multi Item Double-Auction

a. Random halving.

b. Equilibrium calculation.

c. Posted pricing.

d. Random serial dictatorship.

MIDA step a: Random Halving

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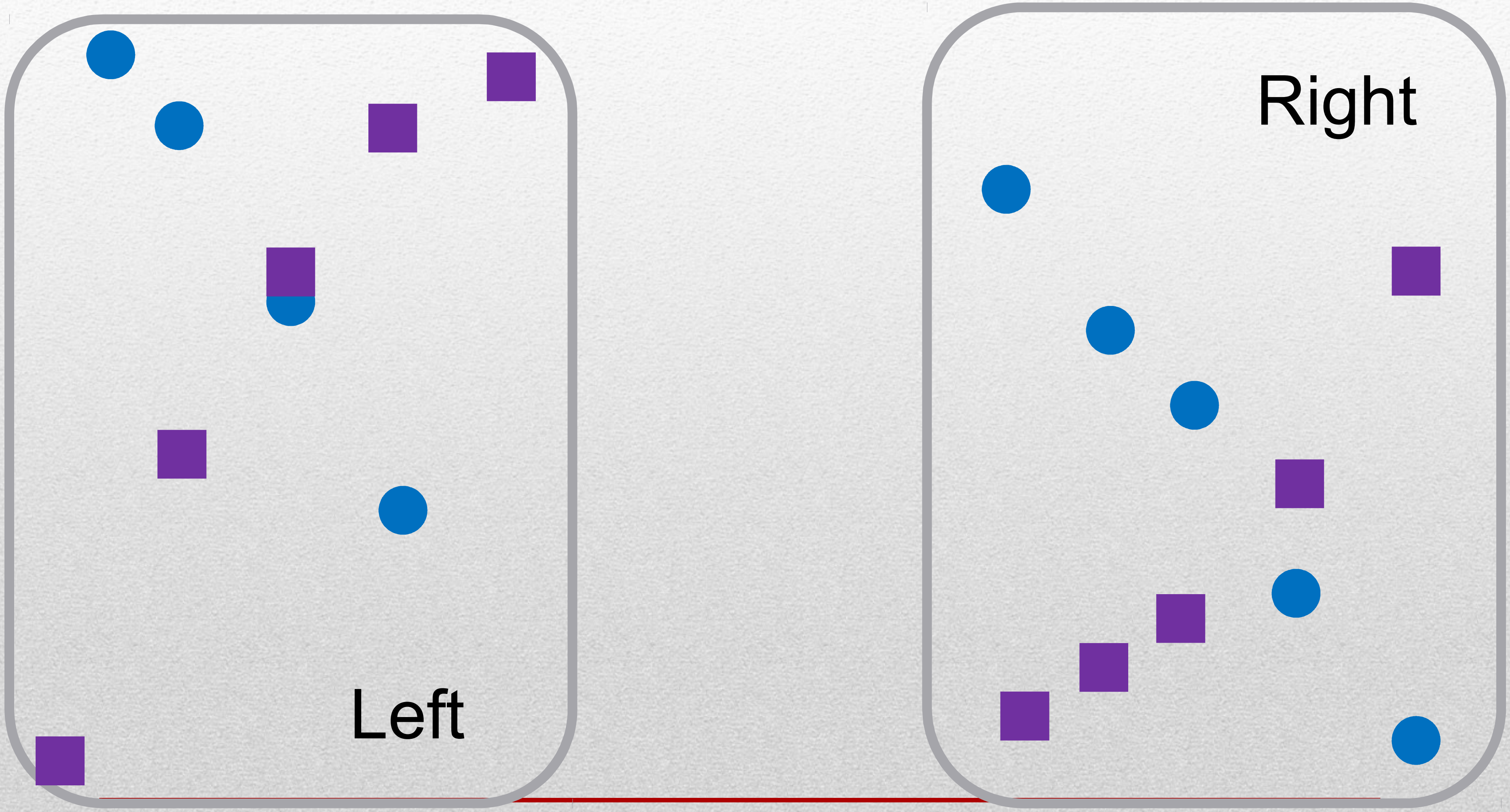
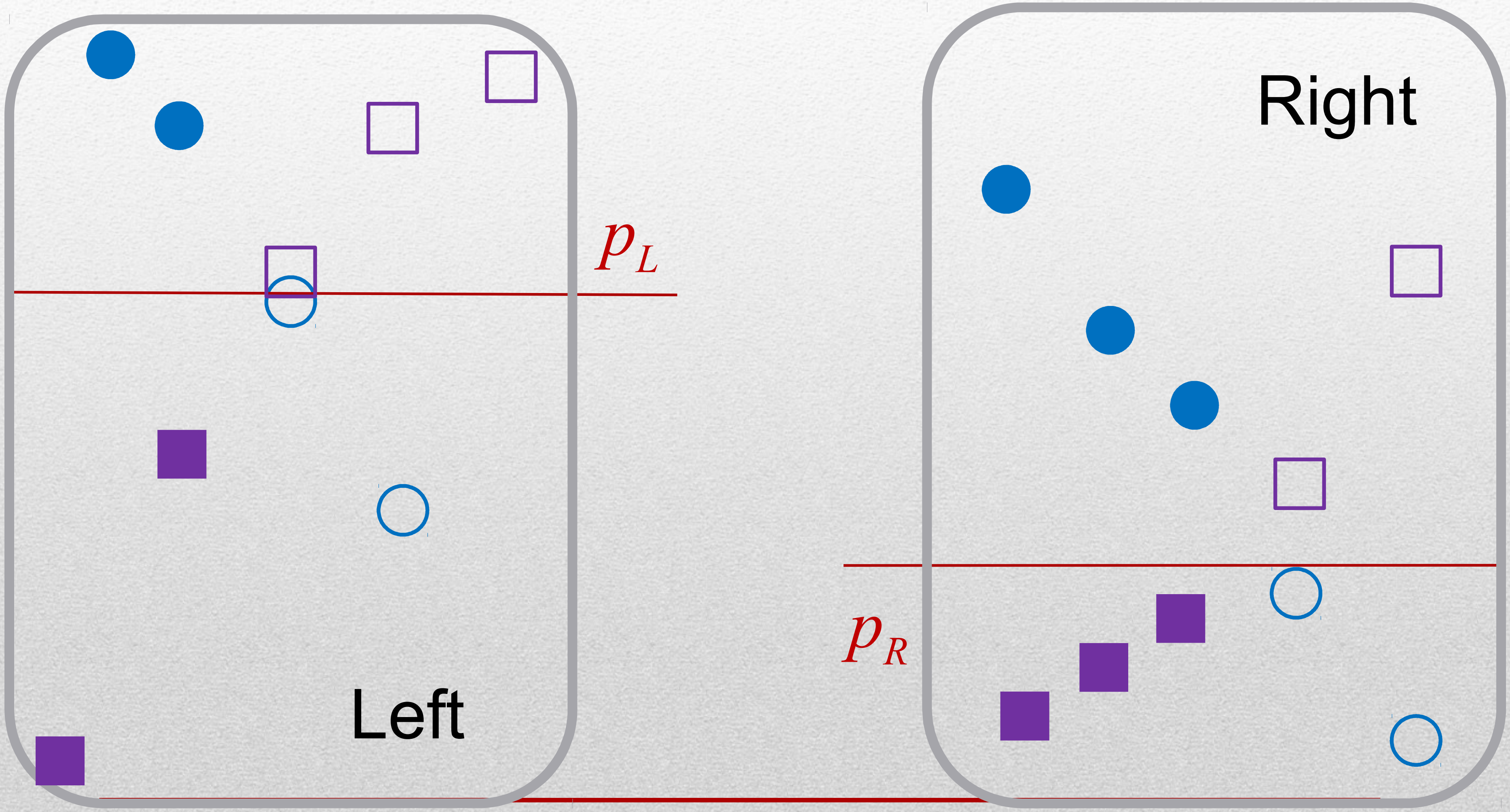
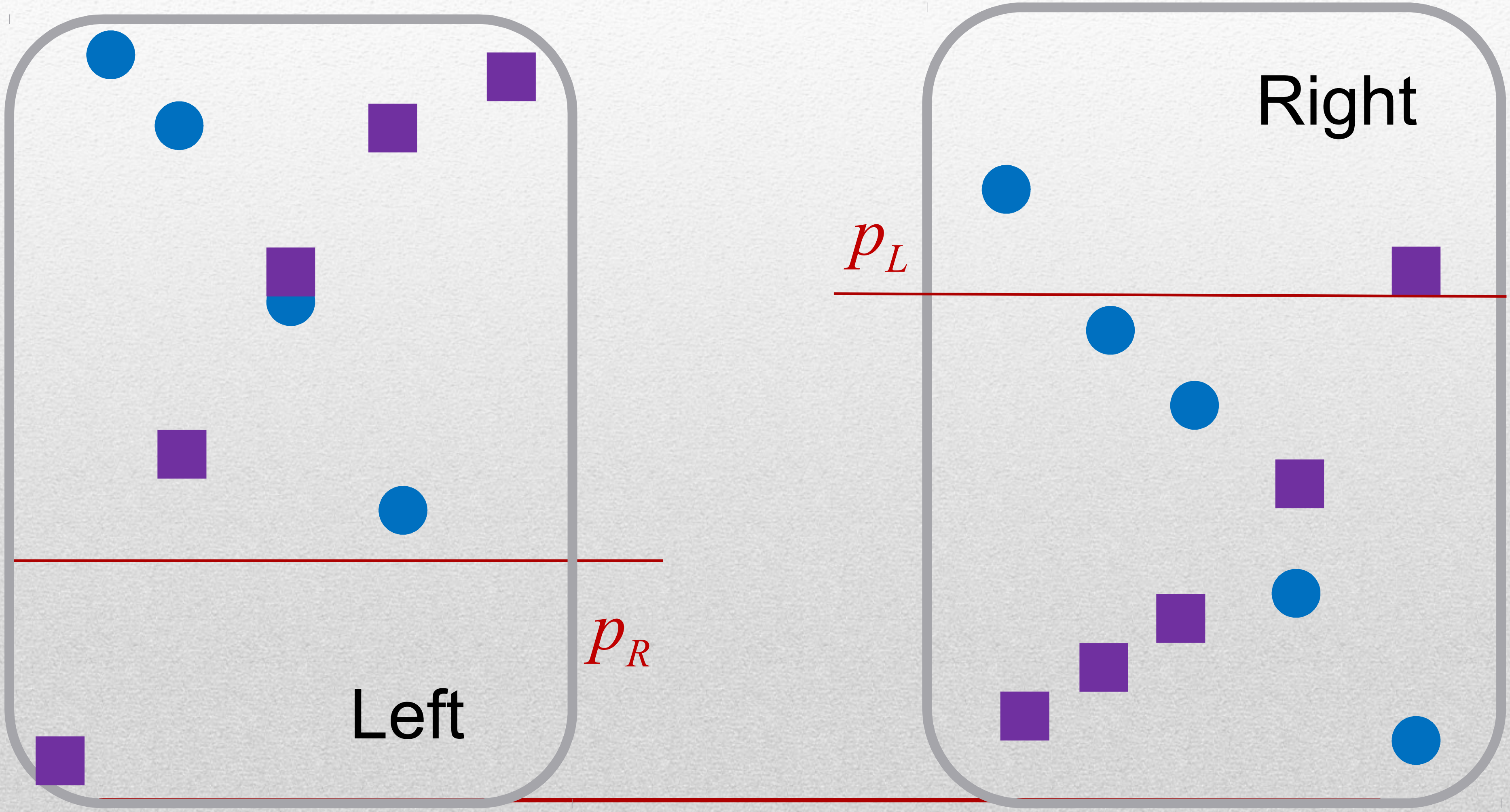


MIDA step b: Equilibrium Calculation

Gross-substitute traders → price-equilibrium exists.

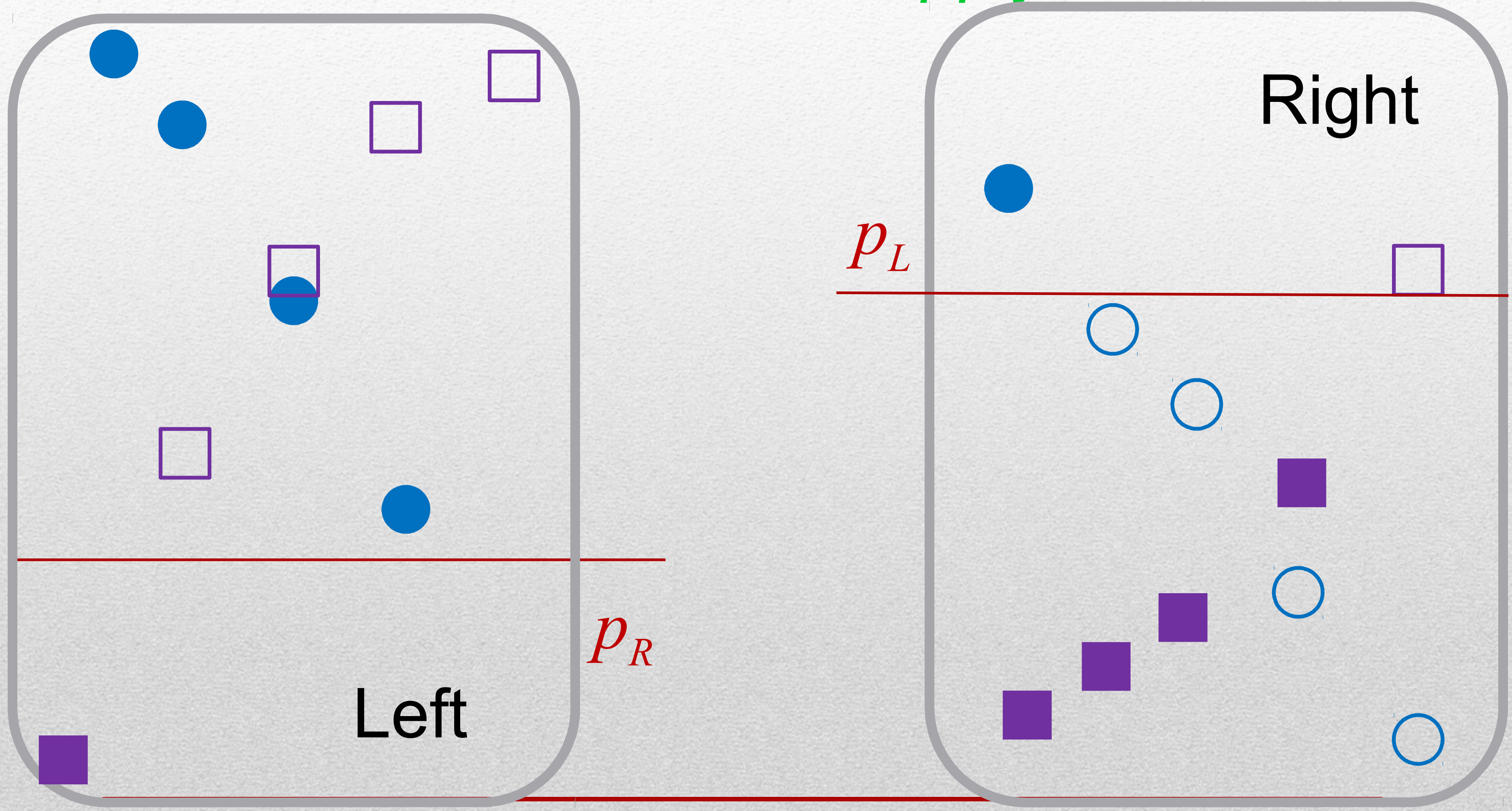


MIDA step c: Posted Pricing



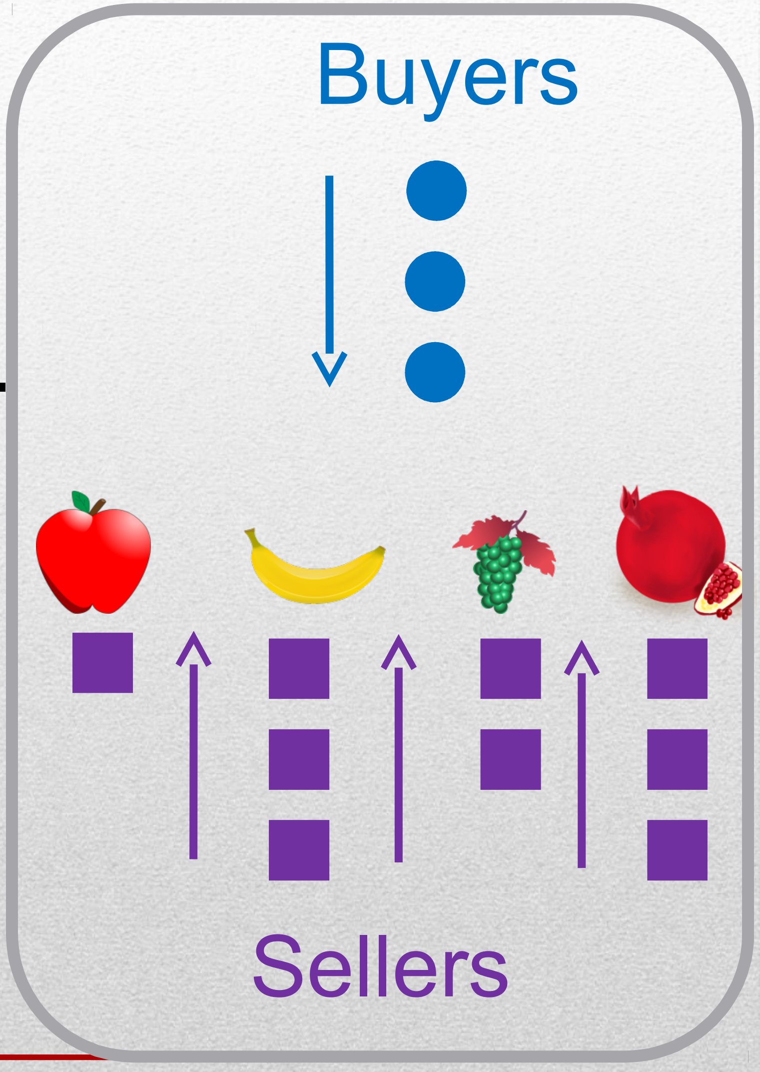
MIDA step d: Random Dictatorship

In case of over-demand/supply – randomize.



MIDA step d: Random Dictatorship

- Order buyers randomly;
- Order sellers randomly;
- First buyer buys from first sellers and goes home.
- Seller goes home when marginal gain < 0.



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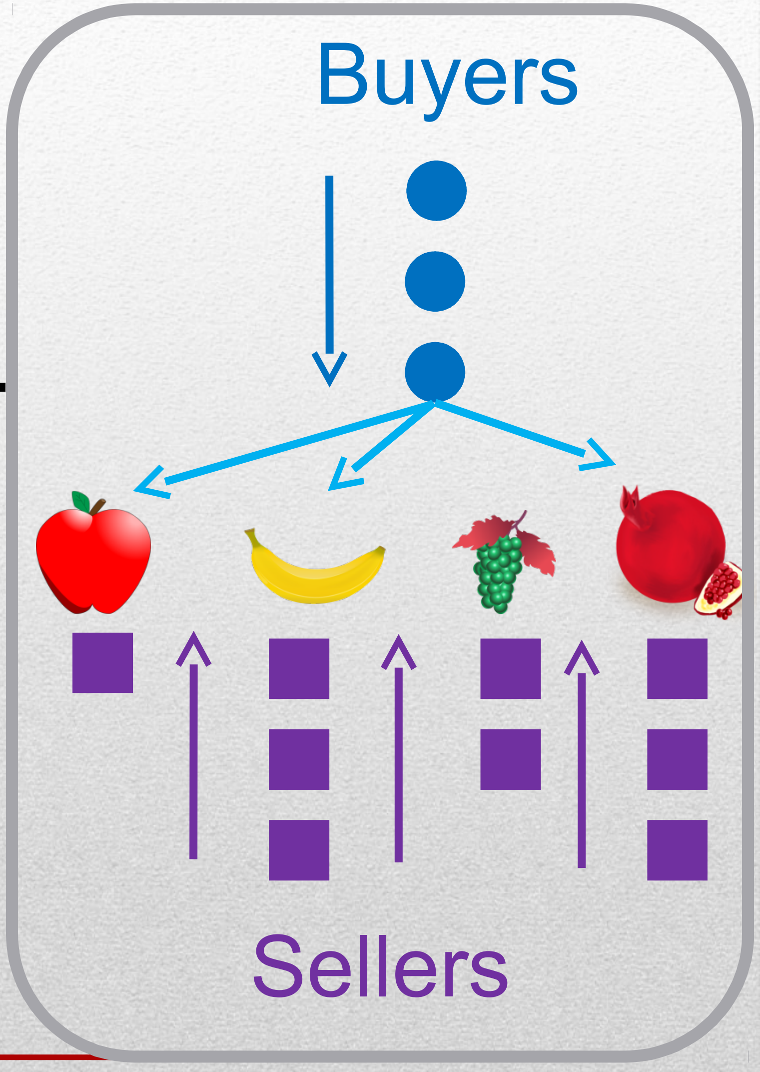

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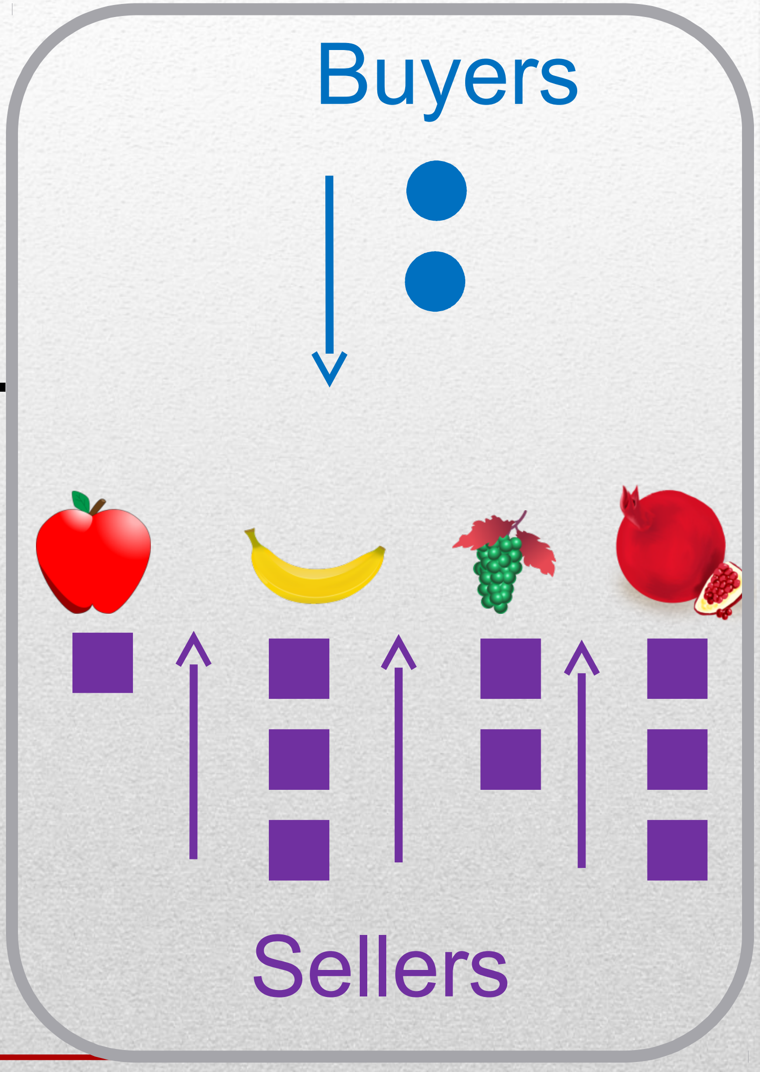

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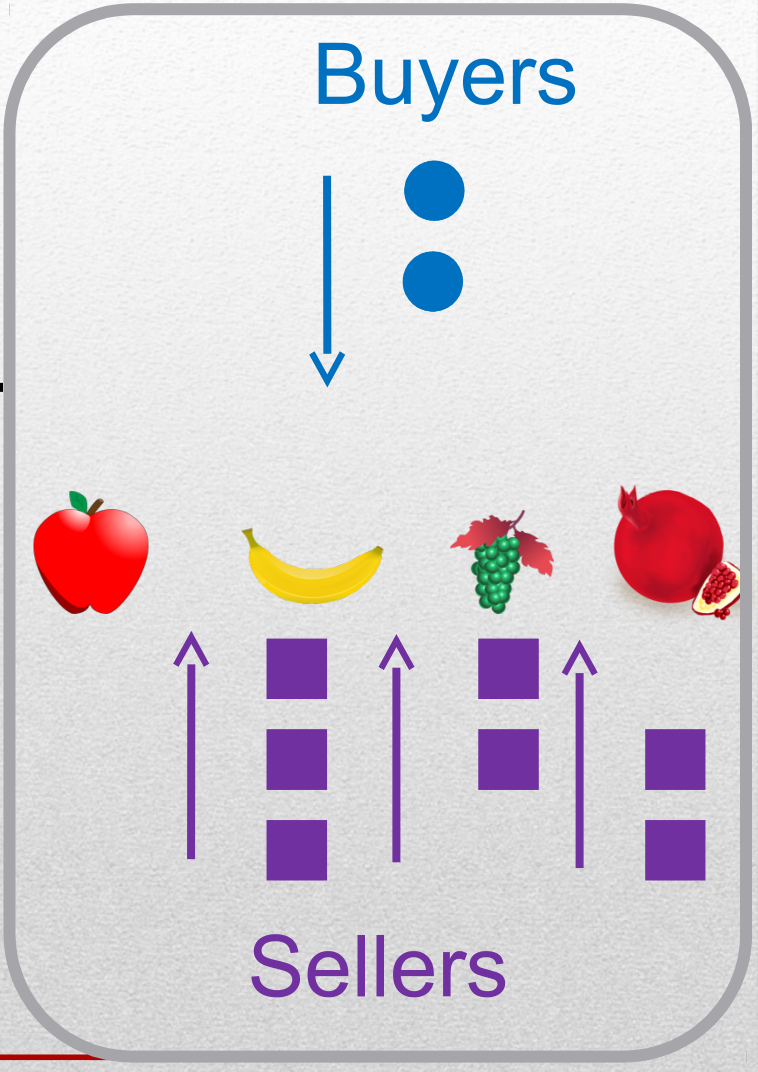

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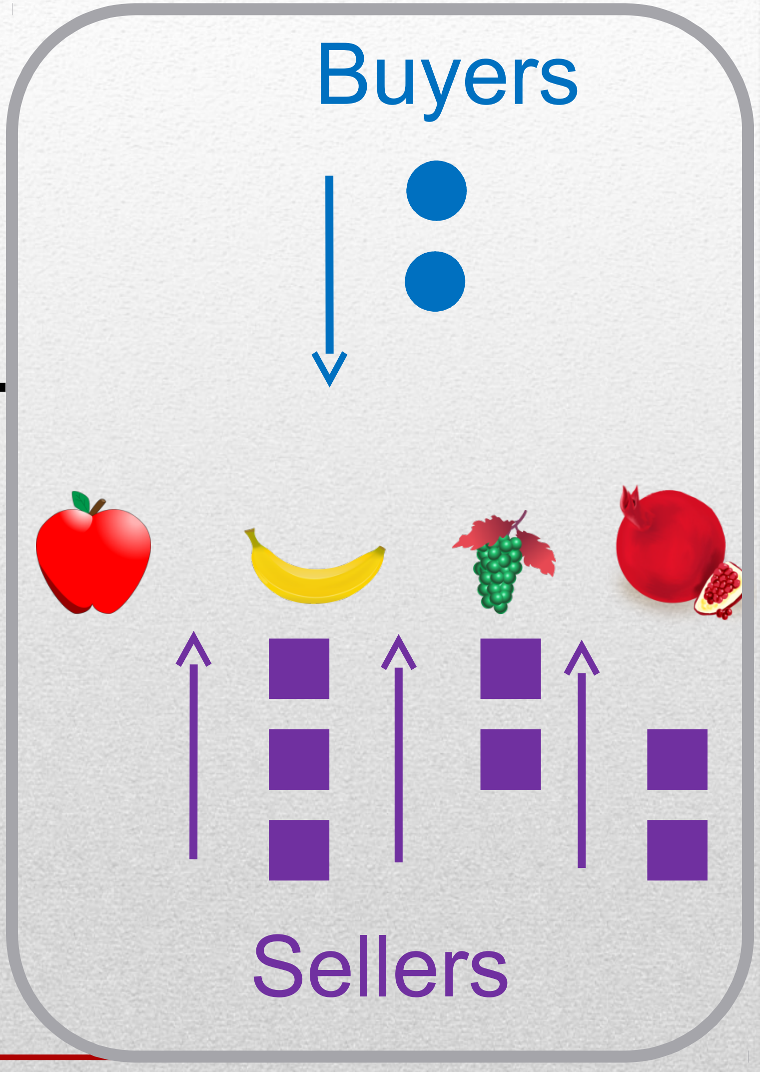

MIDA step d: Random Dictatorship

- Order buyers randomly;
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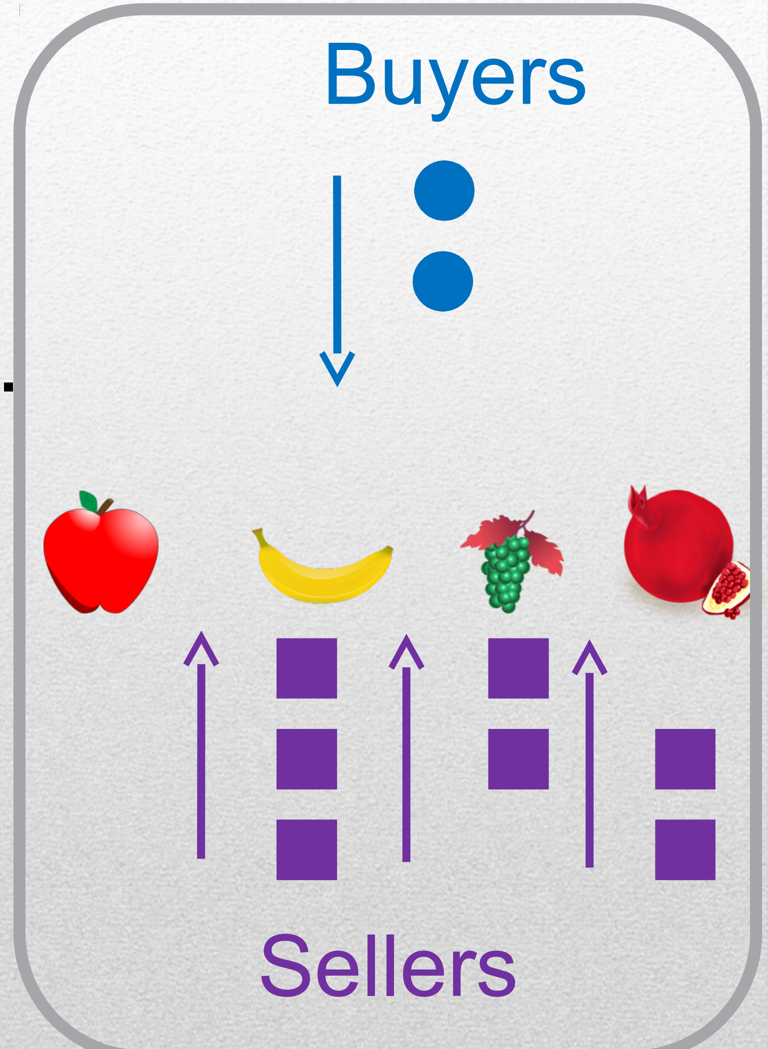
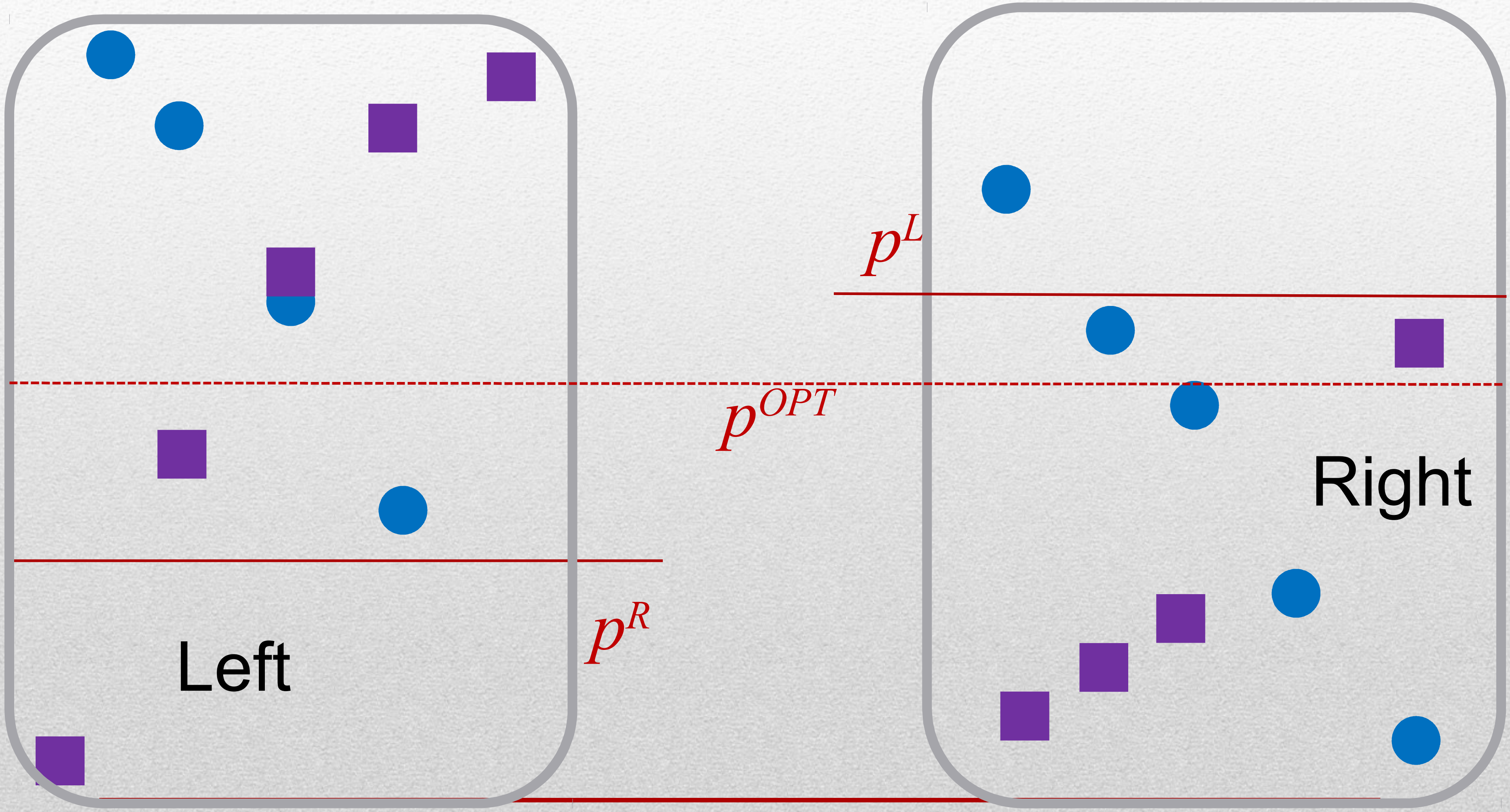
Theorem: If each seller sells one item-type and has decreasing-marginal-gains, then MIDA is truthful.



MIDA:
Estimating the gain-from-trade

Four ways to lose gain-from-trade



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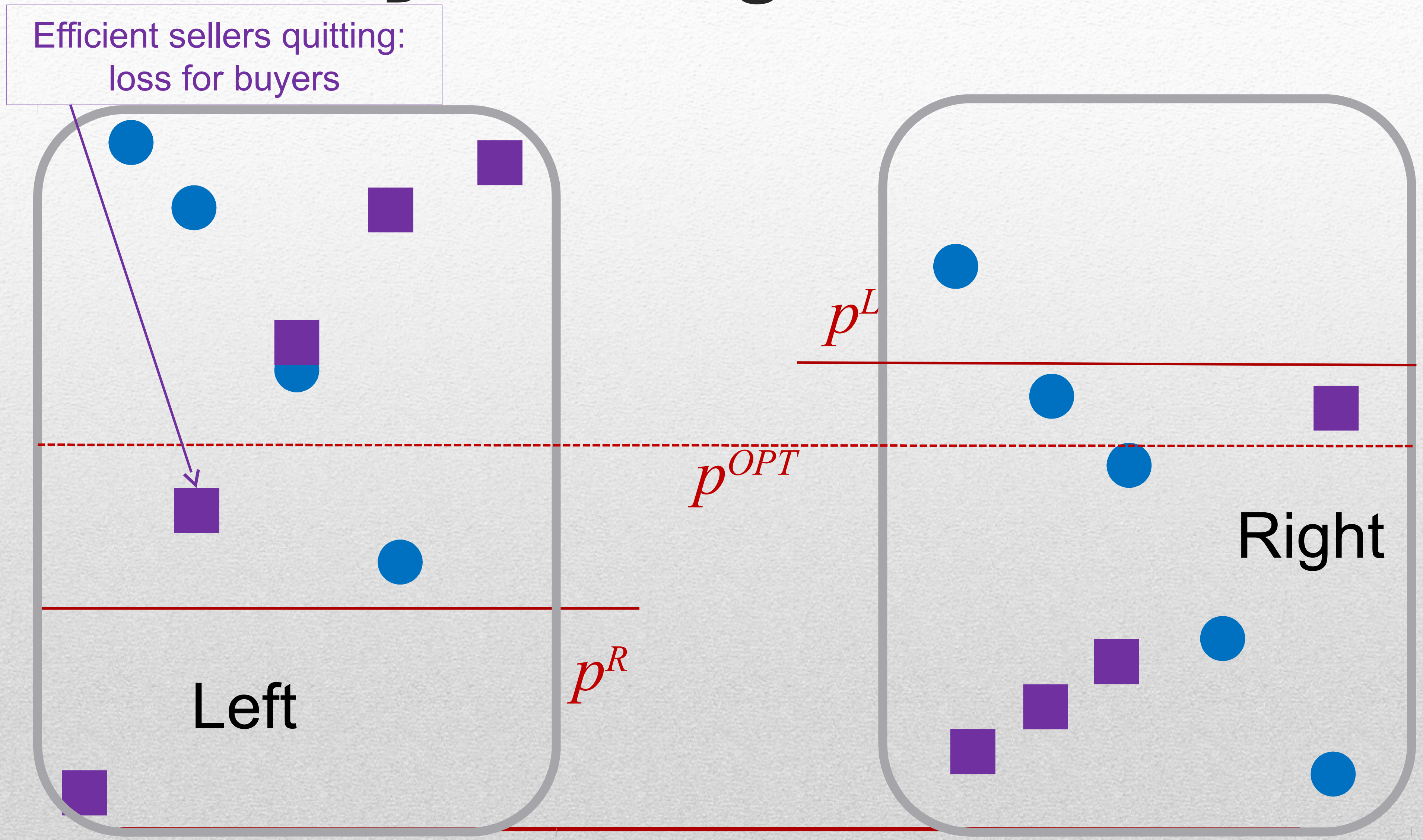

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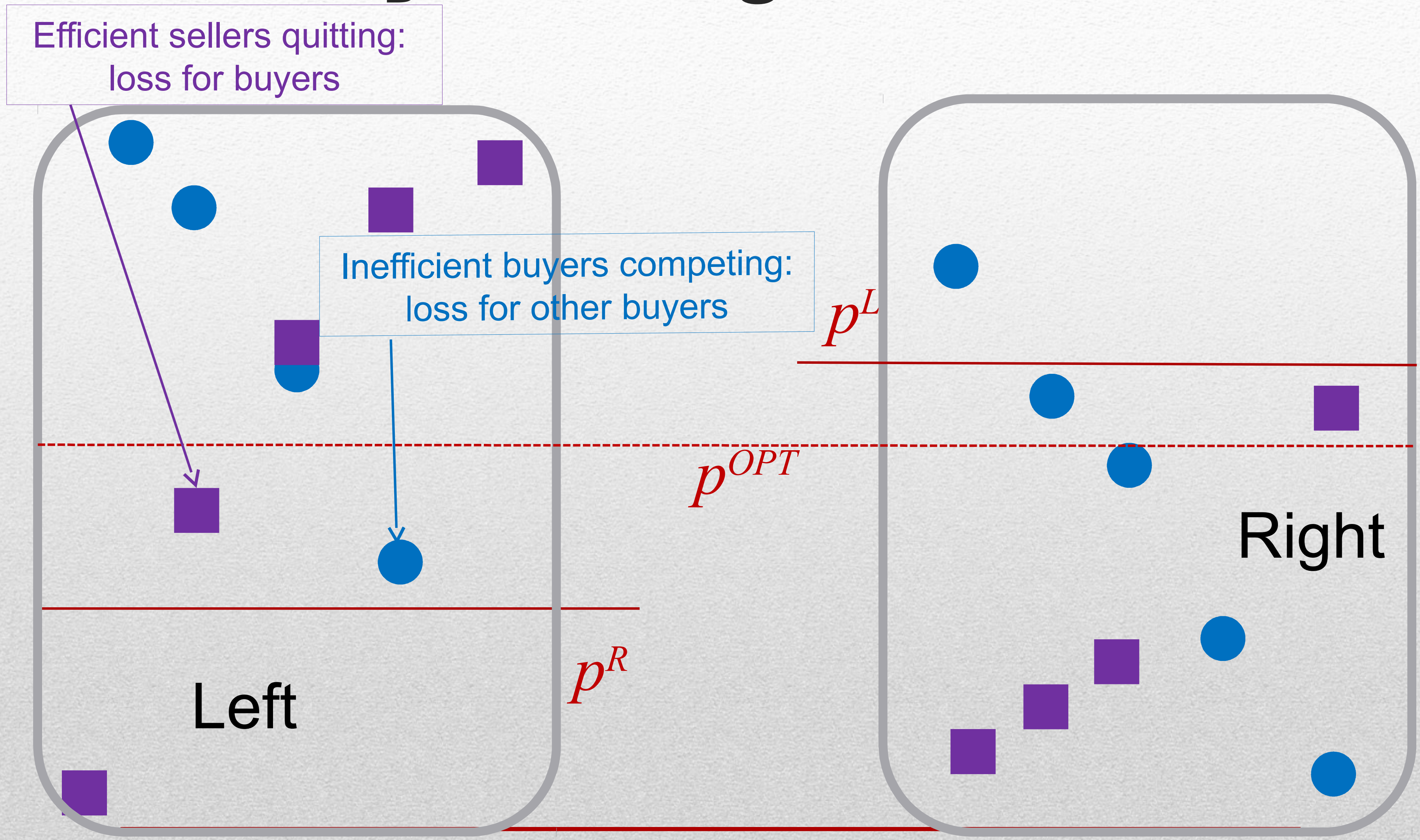

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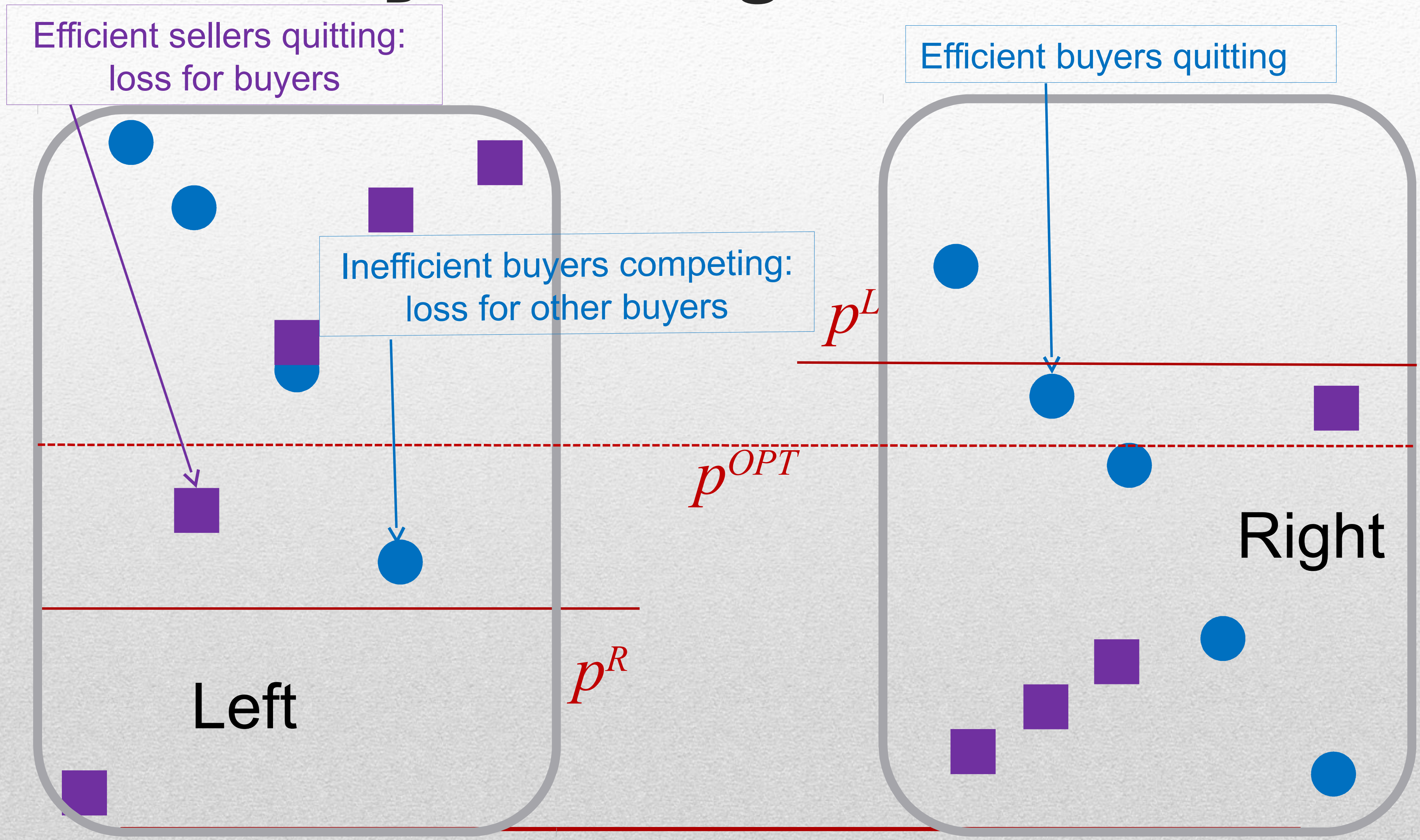

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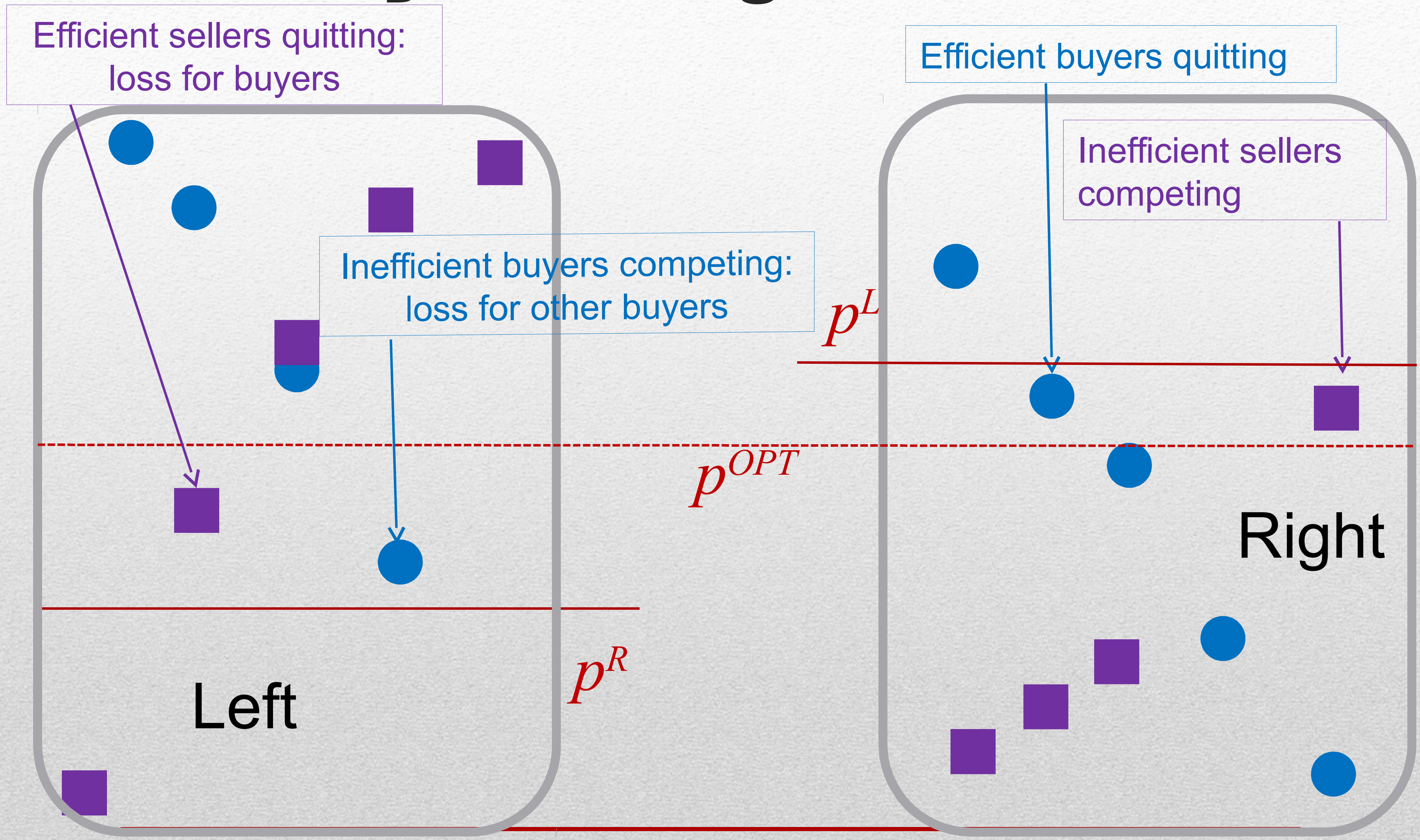

Four ways to lose gain (left market)

For every item-type x, define:

- B_{x*} – buyers who want x in p^{OPT}
- B_{x-} – buyers who want x in p^{OPT} but not in p^{R}
- B_{x+} – buyers who want x in p^{R} but not in p^{OPT}
- S_{x*} – sellers who offer x in p^{OPT}
- S_{x-} – sellers who offer x in p^{OPT} but not in p^{R}
- S_{x+} – sellers who offer x in p^{R} but not in p^{OPT}

We lose $|B_{x-}| + |S_{x+}|$ random sellers and $|S_{x-}| + |S_{x+}|$ random buyers. So:

$$\mathrm{E}[Loss_x] \leq (|B_{x-}| + |B_{x+}| + |S_{x-}| + |S_{x+}|) \ / \ |B_{x*}|$$

Bounding the loss

$$E[Loss_x] \leq (|B_{x-}| + |B_{x+}| + |S_{x-}| + |S_{x+}|) \; / \; k_x$$

Price-equilibrium equations: for every x:

Global population: $|B_{x*}| = |S_{x*}| = k_x$

Right market (*^{R} = the subset sampled to Right*):

$$|B_{x*}^R| + |B_{x+}^R| - |B_{x-}^R| = |S_{x*}^R| + |S_{x+}^R| - |S_{x-}^R|$$

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Concentration bounds: w.h.p:

$$\big|\, |B_{x*}^R| - |B_{x*}|/2 \,\big| < err_x$$

$$\big|\, |S_{x*}^R| - |S_{x*}|/2 \,\big| < err_x$$

$$err_x = m\sqrt{k_x \ln k_x}$$

Bounding the loss

$$\mathrm{E}[Loss_x] \leq (|B_{x-}| + |B_{x+}| + |S_{x-}| + |S_{x+}|) \;/\; k_x$$

Price-equilibrium + Concentration bounds:

With high probability:

$$\big||B_{x-}{}^R| - |B_{x+}{}^R|\big| < 2\, err_x$$

$$\big||S_{x-}{}^R| - |S_{x+}{}^R|\big| < 2\, err_x$$

Bounding the loss

$$E[Loss_x] \leq (|B_{x-}| + |B_{x+}| + |S_{x-}| + |S_{x+}|) \; / \; k_x$$

Price-equilibrium + Concentration bounds:

With high probability:

$$\left| |B_{x-}{}^R| - |B_{x+}{}^R| \right| < 2\, err_x$$

$$\left| |S_{x-}{}^R| - |S_{x+}{}^R| \right| < 2\, err_x$$

Let’s focus on the buyers.

- We **have** bounds on: $\left| |B_{x-}{}^R| - |B_{x+}{}^R| \right|$
- We **need** bounds on: $|B_{x-}|$, $|B_{x+}|$

Bounding the loss: step A

- We **have** bounds: $||B_{x-}^{R}| - |B_{x+}^{R}|| < 2\ err_x$

$||B_{1-}^{R}| - |B_{1+}^{R}|| < 2\ err_1$

$||B_{2-}^{R}| - |B_{2+}^{R}|| < 2\ err_2$

… $||B_{g-}^{R}| - |B_{g+}^{R}|| < 2\ err_g$

- We **derive** bounds on: $|B_{x-}^{R}|$, $|B_{x+}^{R}|$

Bounding the loss: step A

- We **have** bounds: $||B_{x-}{}^{R}| - |B_{x+}{}^{R}|| < 2\ err_x$
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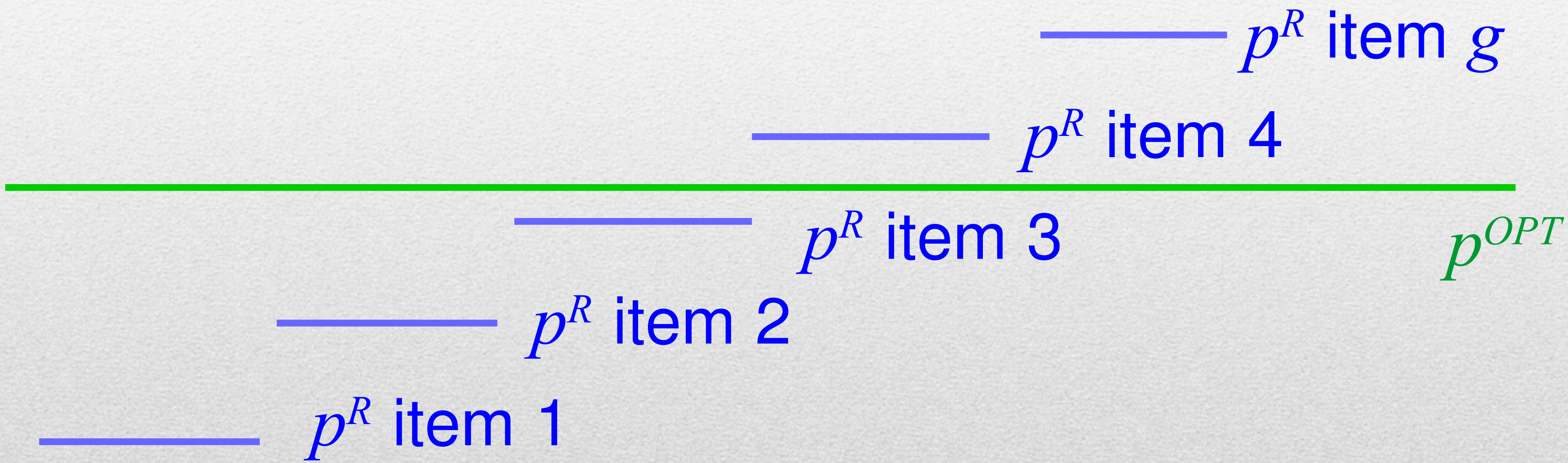
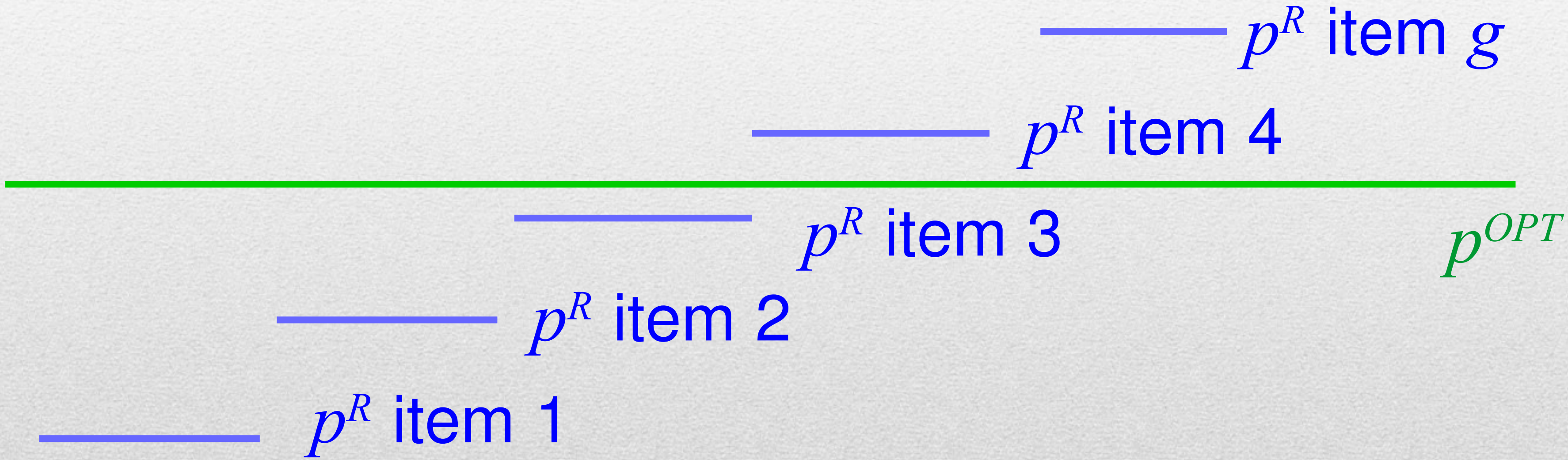


Bounding the loss: step A

- We **have** bounds: $||B_{x-}^{R}| - |B_{x+}^{R}|| < 2\ err_x$
- We **derive** bounds on: $|B_{x-}^{R}|$, $|B_{x+}^{R}|$



Theorem: The demand of gross-substitute agents moves only downwards (Segal-Halevi et al, 2016).

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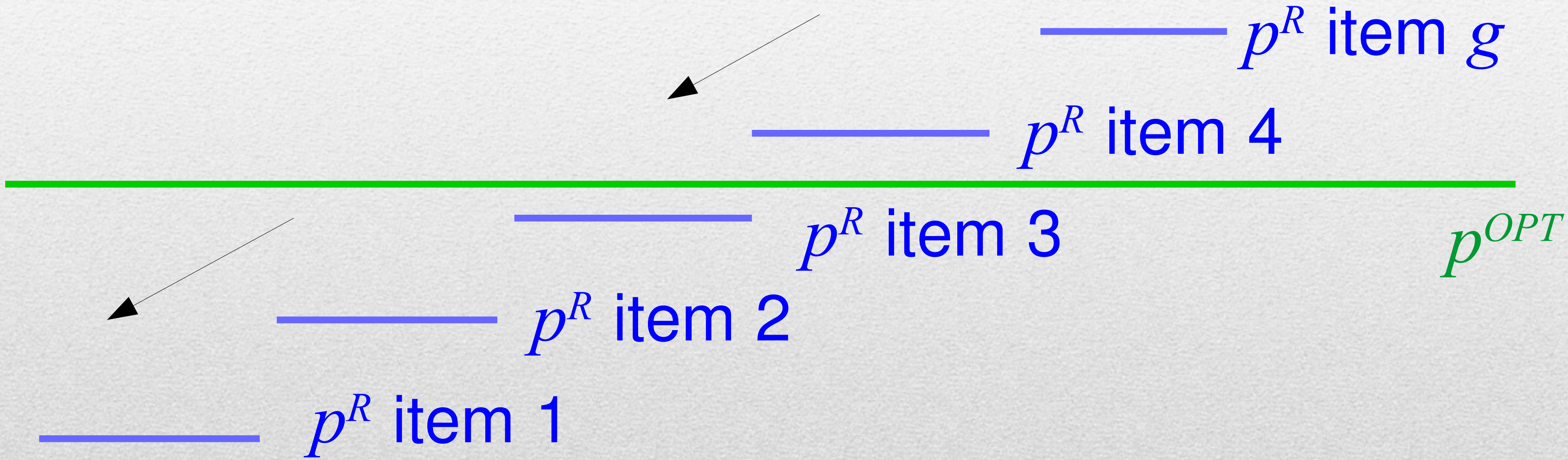
- We **have** bounds: $||B_{x-}{}^{R}| - |B_{x+}{}^{R}|| < 2\ err_x$
- We **derive** bounds on: $|B_{x-}{}^{R}|\ ,\ |B_{x+}{}^{R}|$



Theorem: The demand of gross-substitute agents moves only downwards (Segal-Halevi et al, 2016).

Bounding the loss: step A

- We **have** bounds: $||B_{x-}^R| - |B_{x+}^R|| < 2\ err_x$
- We **derive** bounds on: $|B_{x-}^R|$, $|B_{x+}^R|$

For every item x that became cheaper: $B_{x-}^R \subseteq \cup_{y<x} B_{y+}^R$

- $||B_{1-}^R| - |B_{1+}^R|| < 2\ err_{max}$

  $||B_{2-}^R| - |B_{2+}^R|| < 2\ err_{max}$

  … $||B_{g-}^R| - |B_{g+}^R|| < 2\ err_{max}$
- $|B_{1-}^R| = 0 \rightarrow |B_{1+}^R| < 2\ err_{max}$

  $|B_{2-}^R| < 2\ err_{max} \rightarrow |B_{2+}^R| < 4\ err_{max}$

  …$|B_{g-}^R| < 2^g\ err_{max}$, $|B_{g+}^R| < 2^g\ err_{max}$

Bounding the loss: step B

- We **have** a bound: $|B_{x-}{}^{R}|$, $|B_{x+}{}^{R}| < 2^{g}\, err_{max}$
- We **need** a bound on: $|B_{x-}|$, $|B_{x+}|$

- When T is a **deterministic set** – (like B_{x*}) – determined **before** randomization –

$$\text{w.h.p:} \qquad \big||T^R| - |T|/2\big| < \sqrt{|T| \ln |T|}$$

B_{x-} and B_{x+} are **random sets** - depend on price – determined **after** randomization!

Our solution: bound the **UI dimension** of B_{x-}, B_{x+}

UI Dimension of Random Sets

UI Dimension – property of a random-set.

If **UIDim(*T*) ≤ *d*** then (Segal-Halevi et al, 2017):

$$\text{w.h.p:} \qquad \Big||T^R| - |T|/2\Big| < d \cdot \sqrt{|T| \ln |T|}$$

1. **Containment-Order Rule**: If the support of T is ordered by containment, then UIDim(T) ≤ 1.
2. **Union Rule**:

$\text{UIDim}(T_1 \cup T_2) \leq \text{UIDim}(T_1) + \text{UIDim}(T_2)$

3. **Intersection Rule**: If $|T_1| < t$ then:

$\text{UIDim}(T_1 \cap T_2) \leq \log(t) * (\text{UIDim}(T_1) + \text{UIDim}(T_2))$

Bounding the loss: step B

- We **have** a bound: $|B_{x-}{}^{R}|, |B_{x+}{}^{R}| < 2^{g}\, err_{max}$
- We **derive** a bound on: $|B_{x-}|$, $|B_{x+}|$

Lemma: For every item-type x:

$$B_{x-} = B_{x*} \cap \bigcap_{X \ni x} \left(\bigcup_{Y \not\ni x} \mathbb{B}_{X \prec Y} \right) \implies \mathrm{UIDim}(B_{x-}) \leq 2^{2g} \ln k_{\max}$$

Similarly: $\mathrm{UIDim}(B_{x+}) \leq 2^{2g} \ln k_{\max}$

Corollary: When $k_{max} >> 2^{3g}$, w.h.p:

$$|B_{x-}|, |B_{x+}| < 3 * (2^{g}\, err_{max})$$

Bounding the loss: step C

- We **have** a bound: $|B_{x-}|, |B_{x+}| < 3*2^g *err_{max}$
- Similarly: $|S_{x-}|, |S_{x+}| < 3*2^g *err_{max}$

- Lost deals in item x: $< 12*(2^g err_{max})$
- Lost gain in item x $< 12*(2^g err_{max}) / k_x$
- Lost gain overall $< 12*(2^g err_{max}) / k_{min}$
- Lost gain overall $< Const * o(k_{max}) / k_{min}$

Theorem: Under large-market assumptions, gain-from-trade of MIDA approaches maximum.

Prior-Free Double-Auctions

	Tru	Gain	Agents
Equilibrium	No	1	Multi-parametric (Gross-substitute)
McAfee family	Yes	1-o(1)	Single-parametric / Single-item-type
MIDA	Yes	1-o(1)	Multi-parametric *(Sellers: 1 type, Buyers: g types, Gross-substitute)*.

Acknowledments

- Game theory seminar in BIU
- Ron Peretz
- Simcha Haber
- Tom van der Zanden
- Assaf Romm
- Economic theory seminar in HUJI
- Econ.&Comp. seminar in HUJI
- Algorithms seminar in TAU

Thank you!