



The Rental Harmony Problem

Input: n rooms, fixed total rent R , n tenants with different preferences.

- **Preferences** = function: price-vector \rightarrow set of "best rooms" given prices.

Output: A matching of tenants to rooms, and a price-vector with sum = R .

- **Envy-free (EF)** = each tenant gets a room from his "best rooms" set.

Two Incompatible Preference Models

Model 1: Quasilinear Tenants

The "best rooms" of agent i are
 $\arg \max_{j=1..n} \text{value}_i(j) - \text{price}(j)$

- EF exists by LP duality (Gale, 1960).

Example:

$\text{val}(1)=70, \text{val}(2)=\text{val}(3)=\text{val}(4)=10$

$p=(50,40,10,0) \rightarrow$ best room = 1

Incompatible with miserly tenants

Model 2: Miserly Tenants

If there is a room with price ≤ 0 , then every tenant has a "best room" with price ≤ 0 .

- EF exists by Sperner's Lemma (Su, 1999).

Example:

$p=(10,20,20,50) \rightarrow$ best room = 1

$p=(10,20,40,30) \rightarrow$ best room = 2 (externality)

Incompatible with quasilinear tenants

Extensions:

- Minimum discount (Haake&Raith&Su, 2002).
- Non-negative prices (Sung&Vlach, 2004).
- Max-min utility (Gal&Mash&Procaccia&Zick, 2017).
- Limited budget (Procaccia&Velez&Yu, 2018).

Extensions:

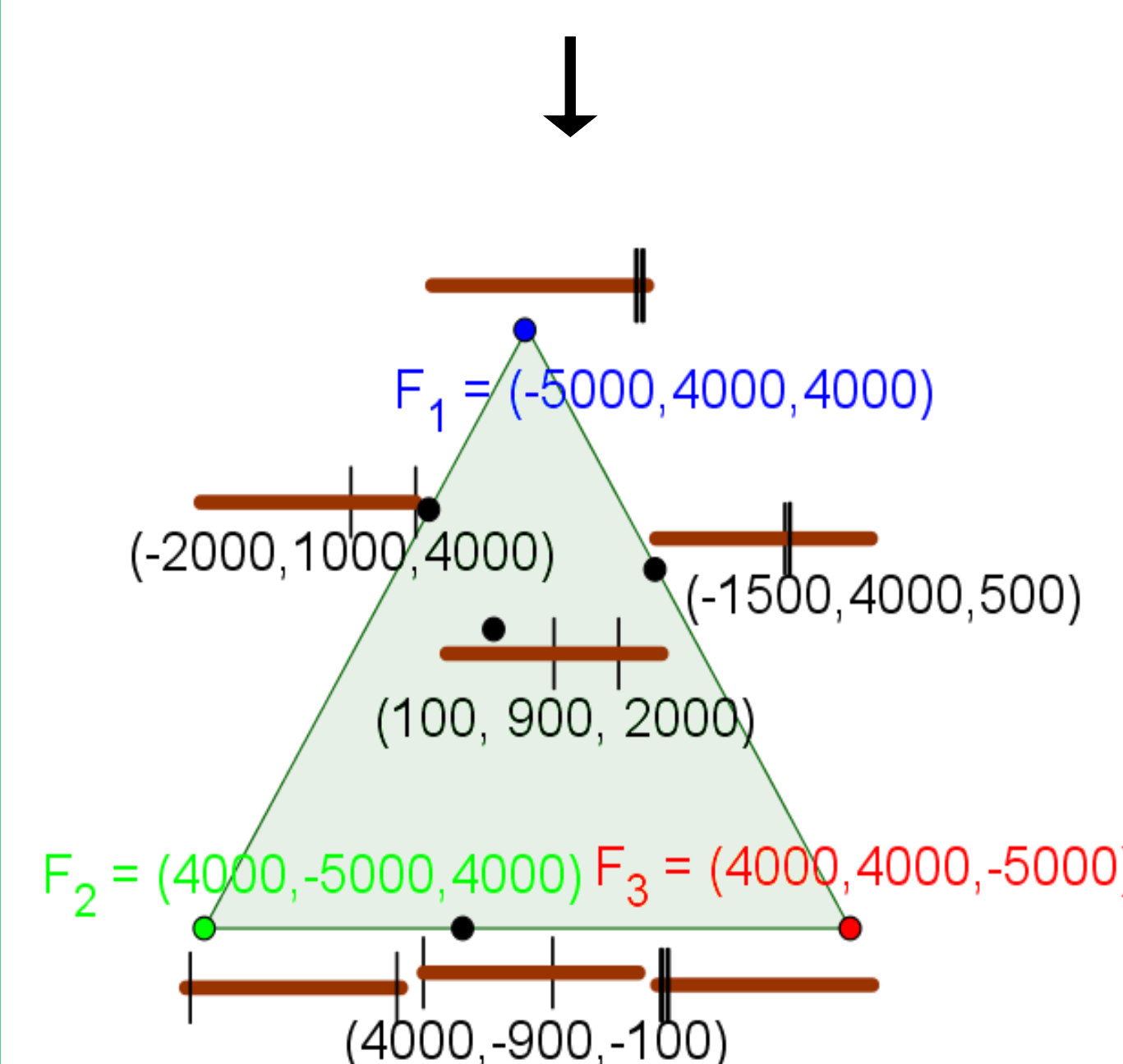
- Roommates (Azrieli&Shmaya, 2014).
- Secretive agent (Frick&Houston&Meunier, 2019).
- Extra agent (Meunier&Su, 2019).
- Multiple houses (Nyman&Su&Zerbib, 2020).

The New Model: Compensable Tenants

For some T , if there is a room with price ≤ 0 and a room with price $\geq T$, then every tenant has a best room with price $< T$.

- Strictly generalizes both models.
- Envy-free matching always exists. Proof idea:
 - Consider the standard simplex $\{(x_1, \dots, x_n) \mid x_1 + \dots + x_n = 1\}$.
 - To each point, associate prices (p_1, \dots, p_n) : $p_j = T - (Tn - R) \cdot x_j$.
 - Triangulate simplex; apply Sperner's lemma; find colorful simplex.
 - Using continuity, shrink the colorful simplex to a single point p .

Example with $R=3000, T=4000$



Corollary: Model 2 extensions are valid for compensable & quasilinear tenants.

Open question: can we generalize the preference model even further?
What is a necessary condition for the existence of Rental Harmony?